* 1. }

So S is subset of N and S is infinite.

Since N is countably infinite and S is infinite subset of N, and we know that infinite subset of a countably infinite set is countably infinite.

So S is also countably infinite.

where and

We know that Z+ is countably infinite and the function B1 🡪 Z is a bijection. So B1 is also countably infinite. Same thing goes for B2.

We know that countable U countable is also countable.

is countable and Z+ is countable infinite,

Therefore, B is countable infinite.

1. Assuming that B is not uncountable, then B is finite set. And since A is a subset of B, therefore A is also a finite set. Because element in A must be in B. So, if B is countable, A is also countable. By contrapositive, it’s true that if A is uncountable, then B is uncountable.
2. is uncountable.

Suppose for contradiction, is countable.

Where ,

Let

Some , so where B =

There are 2 cases:

Case 1:

by definition of B which contradicts

Case 2:

which contradicts

So is uncountable.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | And so on |
| 0 |  | 0.9 | 0.99 | 0.999 | … |
| 1 | 9 | 9.9 | 9.99 | 9.999 | … |
| 2 | 99 | 99.9 | 99.99 | 99.999 | … |
| 3 | 999 | 999.9 | 999.99 | 999.999 | … |
| And so on | … | … | … | … | … |

Therefore, this set is countable as it can be counting off diagonally.

* 1. Suppose S is uncountable set of real numbers [0, 1] where decimal representation consisting of 8’s and 9’s

0.888 where there’s infinite number of 8 after the decimals, same goes to 0.999

By contradiction, suppose S is countable where S it has element s1, s2, s3 and so on.

//not complete//

* 1. Let A = set of R numbers of (0, 1] and B = set of R numbers of (0, 1)

A – B = {1}

Therefore A-B is a finite set.

* 1. Let A = set of R number (0, 1) union Z+ and B = set of R number of (0, 1)

A – B = Z+

Therefore, A – B is countably infinite.

* 1. Let A = set of R number (0, 1) and B = set of R numbers (1, 2)

A – B = A as there are no common elements.

Therefore, A – B is uncountably infinite.